

Warm Up #11-4

1. Find the general term, u_n , for the following geometric sequence:

$$u_2 = 6 \quad \& \quad u_5 = 48$$

2. 5, 3, 1, -1, ... find the 10th term

3. 5, 1, 0.2, ... find the 10th term
(give the exact answer)

Arithmetic sequences (AP) have a
COMMON DIFFERENCE, d .

The n th term is given by the following formula.

$$u_n = u_1 + d(n-1)$$

Geometric sequences (GP) have a
COMMON RATIO, r .

The n th term is given by the following formula.

$$u_n = u_1(r)^{n-1}$$

Try to come up with an easy way to do this problem. A little boy named Gauss did. He was sent to the corner because he was misbehaving and told he couldn't come out until he had the answer. It took him only a few seconds, and he didn't have a calculator!

$$1 + 2 + 3 + \dots + 98 + 99 + 100 = 5050$$

$$\begin{array}{r} +100 \\ +99 \\ +98 \\ \hline 101 \end{array} \quad \begin{array}{r} +3 \\ +2 \\ +1 \\ \hline 101 \end{array} \quad \begin{array}{r} +99 \\ +98 \\ +97 \\ \hline 101 \end{array}$$

$$\frac{100(101)}{2} = 50(101)$$

$$S_n = \frac{n(u_1 + u_n)}{2}$$



$$3 + 5 + 7 + \dots + 81 + 83 + 85$$

$$S_n = \frac{n(u_1 + u_n)}{2}$$

← n = ?

How many terms are being added?

$$u_n = u_1 + d(n-1)$$

$$85 = 3 + 2(n-1)$$

$$82 = 2n - 2$$

$$84 = 2n \longrightarrow n = 42$$

Find the sum:

$$S_{42} = \frac{42(3 + 85)}{2}$$

$$= 21(88)$$

$$= 1848$$

Arithmetic **SERIES**: the **sum** of the terms in a sequence.

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$\frac{n}{2}(a_1 + a_1 + d(n-1))$$

$$1 - (-6)$$

$$S_n = \frac{n}{2}(2a_1 + d(n-1))$$

You try: $-6 + 1 + 8 + 15 + \dots + 141$

$$141 = -6 + 7(n-1)$$

$$147 = 7n - 7$$

$$\frac{154}{7} = \frac{7n}{7}$$

$$n = 22$$

$$S_{22} = \frac{22}{2}[2(-6) + 7(22-1)]$$

$$S_{22} = 11(-12 + 147)$$

$$= 11(135)$$

$$= 1485$$

Geometric Series: The sum of a geometric sequence.

$$S_n = g_1 + g_1r + g_1r^2 + g_1r^3 + \dots + g_1r^{n-3} + g_1r^{n-2} + g_1r^{n-1}$$

$$- (r S_n = g_1r + g_1r^2 + g_1r^3 + \dots + g_1r^{n-3} + g_1r^{n-2} + g_1r^{n-1} + g_1r^n)$$

$$S_n - rS_n = g_1 - g_1r^n$$

$$S_n(1 - r) = g_1(1 - r^n)$$

$$S_n = \frac{g_1(1 - r^n)}{1 - r} \quad \text{or} \quad \frac{g_1(r^n - 1)}{r - 1}$$

Find: $3 + 6 + 12 + 24 + \dots + 1536$ $n = ?$

first find n:

$$S_n = \frac{g_1(r^n - 1)}{r - 1}$$

$$u_n = u_1 r^{n-1}$$

$$\frac{1536}{3} = \frac{3(2)^{n-1}}{3}$$

$$S_{10} = \frac{3(2^{10} - 1)}{2 - 1}$$

$$512 = 2^{n-1}$$

$$S_{10} = 3(2^{10} - 1)$$

$$2^9 = 2^{n-1}$$

$$S_{10} = 3069$$

$$9 = n - 1$$

$$\boxed{n = 10}$$

Last Example: Find the sum to 12 terms

$$2 + 6 + 18 + 54 + \dots = 531,440$$

$$S_{12} = \frac{2(3^{12} - 1)}{3 - 1}$$

$$3^{12} - 1$$

Classwork:

5E.1 p. 141, # 1, 2, 4be, 5, 7, 8b

5E.2 p. 144, # 1, 2abc, 3bc, 6ab

Staple up!

Week 11 Classwork

Warm Up

Blue WS (Tangents & Normals)

5D.1 p.136, # 1-3, 5, 6, 8, 9

HW: IB Exam Practice

(from 2014 Mock Paper 1)

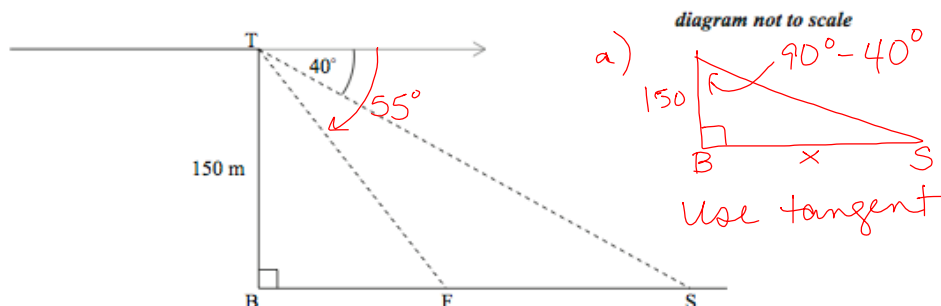
Answers posted on the website.

(Click IB Exam Resources)

Hints for # 2, 5, 8 follow this slide.

Last Quiz: Tuesday, Nov. 21
Sequences and Series

2. Tom stands at the top, T, of a vertical cliff 150 m high and sees a fishing boat, F, and a ship, S. B represents a point at the bottom of the cliff directly below T. The angle of depression of the ship is 40° and the angle of depression of the fishing boat is 55° .

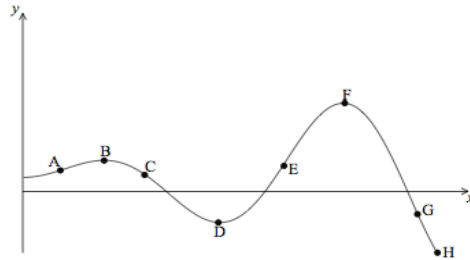


- (a) Calculate, SB, the distance between the ship and the bottom of the cliff. [2 marks]

- (b) Calculate, SF, the distance between the ship and the fishing boat. Give your answer correct to the nearest metre. [4 marks]



5. Consider the graph of the function $y = f(x)$ defined below.



Write down **all** the labelled points on the curve

- (a) that are local maximum points; *← compared to points near it* [1 mark]
 (b) where the function attains its least value; *lowest outcome: H* [1 mark]
 (c) where the function attains its greatest value; *highest outcome: F* [1 mark]
 (d) where the gradient of the tangent to the curve is positive; [1 mark]
 (e) where $f(x) > 0$ and $f'(x) < 0$. [2 marks]

which points have positive outcomes?

which points have negative slopes?

the intersection set!

draw tangents at each point and see where they have a positive slope.

8. Members of a certain club are required to register for one of three sports, badminton, volleyball or table tennis. The number of club members of each gender choosing each sport in a particular year is shown in the table below.

A χ^2 (Chi-squared) test at the 5% significance level is used to determine whether the choice of sport is independent of gender. *← H_0*

	Badminton	Volleyball	Table tennis	Totals
Male	40	20	10	70
Female	20	15	15	50
Totals	60	35	25	120

- (a) Find the expected number of female volleyball players under this hypothesis. [2 marks]
 (b) Write down the p -value for the test. [2 marks]
 (c) State, with a reason, the conclusion of the test. [2 marks]

a) $\frac{\# \text{ females}}{\text{total \#}} \cdot \frac{\# \text{ VB players}}{\text{total \#}} \cdot (\text{total \# people})$

$$\frac{50}{120} \cdot \frac{35}{120} \cdot 120 =$$

b) \rightarrow enter table values in your grapher (2 x 3 matrix)
 Do χ^2 test & write down p value.
 compare it to 5% significance level.